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Bi-maximal Neutrino Mixings And Proton Decay In $SO(10)$ With Anomalous Flavor $\mathcal{U}(1)$

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By supplementing supersymmetric $SO(10)$ with an anomalous $\mathcal{U}(1)$ flavor symmetry and additional 'matter' superfields carrying suitable $\mathcal{U}(1)$ charges, we explain the charged fermion mass hierarchies, the magnitudes of the CKM matrix elements, as well as the solar and atmospheric neutrino data. We stress bi-maximal vacuum neutrino mixings, and indicate how the small or large mixing angle MSW solution can be incorporated. The $\mathcal{U}(1)$ symmetry also implies that $\tau_{p\to K\nu}[SO(10)] \sim (10-100)\cdot \tau_{p\to K\nu}[\text{minimal } SU(5)]$.

The possible existance of neutrino oscillations indicated by the solar and atmospheric neutrino data [1] has inspired many recent attempts to incorporate the phenomena within a variety of theoretical frameworks [2]-[7]. Of course, realistic scenarios should also shed light on the charged fermion masses and their mixings, and we will attempt to do that in this letter. Our main focus is to realize bi-maximal neutrino mixings [2]- [5] within the $SO(10)$ framework, generalizing our earlier work based on $SU(5)$ [3]. The mechanism we propose differs from those suggested earlier for $SO(10)$ [4], [5] and turns out to have important consequences for proton decay as well. An important role in our considerations is played by an anomalous flavor $\mathcal{U}(1)$ symmetry [8], which helps realize the mechanism of maximal neutrino mixings [7]. We also indicate how, if future data requires it, the small or large mixing angle MSW solutions [9] can be incorporated.

For the symmetry breaking $SO(10)$ to $SU(3)_c\times SU(2)_L\times U(1)_Y \equiv G_{321}$, the 'minimal' higgs system consists of $45+\overline{16}+16$. However, many $SO(10)$ scenarios [10] invoke several 45 or 54-plets, or both. Without studying this issue in detail, we will let Σ denote the 45 and/or 54 multiplets, and $\bar{C}+C$ the $\overline{16}+16$ higgs respectively. Their VEVs are responsible for breaking $SO(10)$ down to G_{321}. We will assume that the doublet fragments which reside in the higgs 10-plet($\equiv H$) are light, while the color triplets in H acquire masses of the order of GUT scale($\equiv M_G\sim 2\cdot 10^{16}$ GeV) (such a desirable doublet-triplet splitting may be achieved through one of the mechanisms suggested earlier [10]). For building a realistic 'matter' sector we extend the higgs sector with an $SO(10)$ singlet S and $\overline{16}+16$-plets$\equiv \bar{C}'+C'$ (these states allow us to select the transformation properties of the various superfields under the additional symmetries, see below, in such a way as to forbid all nondesirable operators in our scenario). The GUT scale VEVs are

$$\langle\bar{C}\rangle=\langle C\rangle\sim\langle\Sigma\rangle\sim\langle S\rangle\sim M_G\equiv M_P\epsilon_G,\quad \epsilon_G\sim 10^{-2}\ , \tag{1}$$

while $\langle\bar{C}'\rangle=\langle C'\rangle=0$, where $M_P\simeq 2.4\cdot 10^{18}$ GeV denotes the reduced Planck scale.

We next turn to the symmetries of the theory. Together with 'matter' parity, we introduce a $\mathcal{Z}_2$ $\mathcal{R}$-symmetry under which the various superfields and superpotential W have the following transformation properties:

$$\left(\Sigma,S,H,\bar{C},C,C'\right)\to-\left(\Sigma,S,H,\bar{C},C,C'\right)\ ,$$
$$\bar{C}'\to\bar{C}'\ ,\qquad W\to -W\ . \tag{2}$$

The $\mathcal{Z}_2$ symmetry is crucial for a 'natural' generation of appropriate mass scales.

For understanding the hierarchies between charged fermion masses and the magnitudes of the CKM matrix elements, we introduce an anomalous $\mathcal{U}(1)$ symmetry which often emerges from strings. The associated Fayet-Iliopoulos term equals [11]

$$\xi\int d^4\theta V_A\ ,\qquad \xi=\frac{g_A^2M_P^2}{192\pi^2}\mathrm{Tr}Q\ . \tag{3}$$

The D_A-term is

$$\frac{g_A^2}{8}D_A^2=\frac{g_A^2}{8}\left(\Sigma Q_a|\varphi_a|^2+\xi\right)^2\ , \tag{4}$$

where Q_a is the 'anomalous' charge of φ_a superfield.

We will introduce an $SO(10)$ singlet superfield X with $\mathcal{U}(1)$ charge $Q_X=1$, whose VEV breaks $\mathcal{U}(1)$. Assuming $\mathrm{Tr}Q<0$ ($\xi<0$), the cancellation of (4) fixes the VEV of the scalar component of X:

$$\langle X\rangle=\sqrt{-\xi}\ . \tag{5}$$

We will assume that

$$\frac{\langle X\rangle}{M_P}\equiv\epsilon\simeq 0.22\ , \tag{6}$$

where the parameter ϵ plays an essential role in our analysis. By exploiting the anomalous $\mathcal{U}(1)$ as a flavor symmetry [8], we will gain a natural understanding of the hierarchies of the charged fermion masses and mixings, whose magnitudes will be expressed in powers of ϵ (under $\mathcal{Z}_2$ symmetry, $X \to X$).

The $\mathcal{U}(1)$ charges of the 'higgs' superfields are assigned to be

$$Q_X = 1 \ , \quad Q_\Sigma = Q_S = 0 \ , \quad Q_H = -5/2$$
$$Q_C = -Q_{\bar{C}} = -15/4 \ , \ \ Q_{\bar{C}'} = -5/4 \ , \ \ Q_{C'} = -15/4 \ . \tag{7}$$

From the superpotential couplings

$$\frac{\Sigma + S}{M_P} H\bar{C}\bar{C}' + \left(\frac{X}{M_P}\right)^5 M_P \bar{C}'C' \ , \tag{8}$$

one can readily verify that the 'light' h_u fully resides in H, while h_d resides in H and C', with 'weights' of order unity and $\frac{\epsilon_G^2}{\epsilon^5} \sim 0.2$ respectively:

$$H \supset (h_u, h_d) \ , \quad C' \gtrsim \frac{\epsilon_G^2}{\epsilon^5} h_d \ . \tag{9}$$

The Yukawa sector, constructed with the minimal set of 'matter' 16_i's ($i = 1, 2, 3$), involve the couplings $16_i 16_j H$, which yield the undesirable asymptotic relations $\hat{m}_U^0 = \hat{m}_D^0 = \hat{m}_E^0$, and a trivial CKM matrix $\hat{V}_{CKM} = \mathbf{1}$. For obtaining a realistic pattern of fermion masses and mixings, we extend the 'matter' sector and introduce three supermultiplets 10_i (which are crucial for removing the $\hat{m}_U^0 = \hat{m}_{D,E}^0$ degeneracy [12], [4]), and two pairs of vector-like states $[\bar{F}(\overline{16}) + F(16)]_{1,2}$ (which help avoid the relation $\hat{m}_D^0 = \hat{m}_E^0$). The transformation properties of the various 'matter' superfields are:

$$\mathcal{U}(1) \ : \quad Q_{16_1} = -7/4 \ , \ \ Q_{16_2} = -3/4 \ , \ \ Q_{16_3} = 5/4 \ ,$$
$$Q_{10_1} = 3/2 \ , \ \ Q_{10_2} = Q_{10_3} = 5/2 \ ,$$
$$Q_{F_1} = -Q_{\bar{F}_1} = Q_{16_1} \ , \ \ Q_{F_2} = -Q_{\bar{F}_2} = Q_{16_2} \ , \tag{10}$$

$$\mathcal{Z}_2 : \left(16_i, 10_i, F_{1,2}, \bar{F}_{1,2}\right) \to -\left(16_i, 10_i, F_{1,2}, \bar{F}_{1,2}\right) . \tag{11}$$

From the couplings

$$\begin{array}{c c} & \begin{array}{ccc} 10_1 & 10_2 & 10_3 \end{array} \\ \begin{array}{c} 16_1 \\ 16_2 \\ 16_3 \end{array} & \left(\begin{array}{ccc} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{array}\right) C \ , \end{array} \tag{12}$$

after substituting the VEV of C, the $\bar{5}_i$ fragments (of $SU(5)$), which reside in 16_i-plets form heavy massive states with 5_{10_i}. Since the couplings $10_i 10_j$ are forbidden, the 'light' states $(d^c, l)_i$ fully reside in 10_i. The remaining 'light' quark-lepton states reside in 16_i. We therefore have

$$10_i \supset (d^c, l)_i \ , \quad 16_i \supset (q, u^c, e^c)_i \ . \tag{13}$$

The couplings

$$\begin{array}{c c} & \begin{array}{ccc} 16_1 & 16_2 & 16_3 \end{array} \\ \begin{array}{c} 16_1 \\ 16_2 \\ 16_3 \end{array} & \left(\begin{array}{ccc} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{array}\right) H \ , \end{array} \tag{14}$$

which respect the transformation properties (2), (7), (10), (11), are responsible for generating masses for the up type quarks. Taking into account (9), (13) yields the desirable hierarchies

$$\lambda_t \sim 1 \ , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^4 : 1 \ . \tag{15}$$

The down quark and charged lepton masses emerge from the couplings

$$\begin{array}{c c} & \begin{array}{ccc} 10_1 & 10_2 & 10_3 \end{array} \\ \begin{array}{c} 16_1 \\ 16_2 \\ 16_3 \end{array} & \left(\begin{array}{ccc} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{array}\right) C' \ . \end{array} \tag{16}$$

Equations (9), (13) yield the desirable hierarchies

$$\lambda_b \sim \frac{\epsilon_G^2}{\epsilon^5} \ , \ \ \lambda_d : \lambda_s : \lambda_b \sim \epsilon^4 : \epsilon^2 : 1 \ , \quad \tan\beta \sim \frac{\epsilon_G^2}{\epsilon^5}\frac{m_t}{m_b} \ ,$$
$$\lambda_\tau \sim \frac{\epsilon_G^2}{\epsilon^5} \ , \ \ \lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^4 : \epsilon^2 : 1 \ . \tag{17}$$

However, the degeneracy $\hat{m}_D^0 = \hat{m}_E^0$ still holds at this stage since the $SU(5)$ symmetry is not broken in (16). To remove this drawback, we invoke the states $(\bar{F} + F)_{1,2}$. The relevant couplings will be

$$\begin{array}{c c} & \begin{array}{ccc} 16_1 & 16_2 & 16_3 \end{array} \\ \begin{array}{c} \bar{F}_1 \\ \bar{F}_2 \end{array} & \left(\begin{array}{ccc} 1 & 0 & 0 \\ \epsilon & 1 & 0 \end{array}\right) (S+\Sigma) \ , \end{array} \quad \begin{array}{c c} & \begin{array}{cc} F_1 & F_2 \end{array} \\ \begin{array}{c} \bar{F}_1 \\ \bar{F}_2 \end{array} & \left(\begin{array}{cc} 1 & 0 \\ \epsilon & 1 \end{array}\right) (S+\Sigma). \end{array} \tag{18}$$

From (18) we verify that the mixings of $(q, u^c, e^c)_{16_{1,2}}$ with $(q, u^c, e^c)_{F_{1,2}}$ are of the same order as the masses of $(\bar{F} + F)_{1,2}$ states. This means that the light states $(q, u^c, e^c)_{1,2}$ remain with 'weights' of order unity in $16_{1,2}$ and $F_{1,2}$ respectively. Note that the Σ field(s) violate $SU(5)$ in (18), and therefore the unwanted asymptotic relations $m_e^0 = m_d^0$, $m_\mu^0 = m_s^0$ are avoided, while the hierarchical structure in (17) is unchanged. Since the states in 16_3 are not affected by (18), $b-\tau$ unification still holds at the GUT scale.

From (14) and (16), for the CKM matrix elements we find

$$V_{us} \sim \epsilon \ , \ V_{cb} \sim \epsilon^2 \ , \ V_{ub} \sim \epsilon^3 \ . \tag{19}$$

Thus, thanks to the anomalous $\mathcal{U}(1)$ flavor symmetry and our choice of extended 'matter' multiplets, one can obtain a realistic pattern for the charged fermion masses and CKM mixings within the framework of SUSY $SO(10)$.

Next we turn to the neutrino sector and attempt to account for the solar and atmospheric neutrino data. We concentrate on the bi-maximal (vacuum) mixing scenario, but later point out how the small (or large) mixing angle MSW oscillations can be realized in the present framework. Since the light fragments of l_i reside in 10_i states, and 10_2 and 10_3 have the same $\mathcal{U}(1)$ charge (see (10)), we can expect naturally large $\nu_\mu - \nu_\tau$ mixing. This also can be seen from the texture in (16). Introducing an $SO(10)$ singlet right handed neutrino $\mathcal{N}_3$ with suitable mass, the state 'ν_3' will gain an appropriate mass relevant for the atmospheric puzzle. At this stage the other two neutrino states are massless. From (16) one can see that large $\nu_e - \nu_{\mu,\tau}$ mixing will not be realized in a straightforward way (expected mixing is of order ϵ).

To obtain large $\nu_e - \nu_{\mu,\tau}$ mixing, we invoke the mechanism suggested in [7], which naturally yields 'maximal' mixings between neutrino flavors. For this purpose we introduce two additional $SO(10)$ singlet states $\mathcal{N}_1$, $\mathcal{N}_2$. The states $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$ have the following transformation properties under $\mathcal{U}(1) \times \mathcal{Z}_2$:

$$\mathcal{U}(1)\ :\ Q_{\mathcal{N}_1} = -Q_{\mathcal{N}_2} = -2\ , \quad Q_{\mathcal{N}_3} = 0\ ,$$
$$\mathcal{Z}_2\ :\quad \mathcal{N}_i \to -\mathcal{N}_i\ . \tag{20}$$

The relevant couplings are

$$W_{\mathcal{N}_3} = \kappa S \mathcal{N}_3^2 + \epsilon(a\epsilon 10_1 + b10_2 + c10_3) H \mathcal{N}_3, \tag{21}$$

$$\begin{array}{c} \\ 10_1 \\ 10_2 \\ 10_3 \end{array}\!\!\begin{array}{c} \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{pmatrix} \epsilon^4 & 1 \\ \epsilon^3 & 0 \\ \epsilon^3 & 0 \end{pmatrix} \end{array} H\ , \qquad \begin{array}{c} \\ \mathcal{N}_1 \\ \mathcal{N}_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{pmatrix} \epsilon^4 & 1 \\ 1 & 0 \end{pmatrix} \end{array} \kappa' S\ , \tag{22}$$

where κ, κ', a, b, c are dimensionless coefficients. Note that there also exists the coupling $\alpha S \epsilon^2 \mathcal{N}_1 \mathcal{N}_3$ which, if properly suppressed (see (24)), will not be relevant.

Let us choose the basis in which the charged lepton matrix (16) is diagonal. This choice is convenient because the matrix which diagonalizes the neutrino mass matrix will then coincide with the lepton mixing matrix. The hierarchical structure of the couplings in (21) will not be altered, while the 'Dirac' and 'Majorana' masses from (22) will respectively have the forms

$$m_D = \begin{pmatrix} \epsilon^4 & 1 \\ \epsilon^3 & \epsilon \\ \epsilon^3 & \epsilon \end{pmatrix} h_u\ , \quad M_R = \begin{pmatrix} \epsilon^4 & 1 \\ 1 & 0 \end{pmatrix} M_P \kappa' \epsilon_G\ . \tag{23}$$

Taking

$$\kappa \sim 10^{-3}\ , \quad \alpha < 2 \cdot 10^{-2} \tag{24}$$

and the other coefficients of order unity, integration of the $\mathcal{N}$ states leads to the following 'light' neutrino mass matrix:

$$\hat{m}_\nu = \hat{A} m + \hat{B} m'\ , \tag{25}$$

where

$$m \equiv \frac{\epsilon^2 h_u^2}{\kappa M_P \epsilon_G}\ , \quad m' \equiv \frac{\epsilon^3 h_u^2}{M_P \kappa' \epsilon_G}\ , \tag{26}$$

$$\hat{A} = \begin{pmatrix} a^2\epsilon^2 & ab\epsilon & ac\epsilon \\ ab\epsilon & b^2 & bc \\ ac\epsilon & bc & c^2 \end{pmatrix} m\ ,$$

$$\hat{B} = \begin{pmatrix} \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \\ 1 & \epsilon & \epsilon \end{pmatrix} m'\ \cdot \tag{27}$$

The 'light' eigenvalues are

$$m_{\nu_3} \simeq m(b^2 + c^2 + a^2\epsilon^2) \sim 6 \cdot 10^{-2}\ \text{eV}\ ,$$
$$m_{\nu_1} \simeq m_{\nu_2} \simeq m' \sim 1.3 \cdot 10^{-5}\ \text{eV}\ . \tag{28}$$

Ignoring CP violation the neutrino mass matrix (25) can be diagonalized by the orthogonal transformations $\nu_\alpha = U_\nu^{\alpha i} \nu_i$, where $\alpha = e, \mu, \tau$ denotes flavor indices, and $i = 1, 2, 3$ the mass eigenstates. U_ν has the form

$$U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s_1 \\ -\frac{1}{\sqrt{2}} c_\theta & \frac{1}{\sqrt{2}} c_\theta & s_\theta \\ \frac{1}{\sqrt{2}} s_\theta & -\frac{1}{\sqrt{2}} s_\theta & c_\theta \end{pmatrix}\ , \tag{29}$$

with

$$\tan\theta = \frac{b}{c}\ , \quad s_1 = \frac{a\epsilon}{\sqrt{b^2 + c^2}}\ , \tag{30}$$

and $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$. From (25)-(30) the solar and atmospheric neutrino oscillation parameters are

$$\Delta m^2_{21} \sim 2m'^2\epsilon \simeq 7 \cdot 10^{-11}\ \text{eV}^2\ ,$$
$$\mathcal{A}(\nu_e \to \nu_{\mu,\tau}) = 1 - \mathcal{O}(\epsilon^2) \simeq 0.9 - 1\ , \tag{31}$$

$$\Delta m^2_{32} \simeq m^2_{\nu_3} \sim 4 \cdot 10^{-3}\ \text{eV}^2\ ,$$
$$\mathcal{A}(\nu_\mu \to \nu_\tau) = \frac{4b^2c^2}{(b^2 + c^2)^2} - \mathcal{O}(\epsilon^2)\ , \tag{32}$$

where the oscillation amplitudes are defined as

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = 4\Sigma_{j<k} U_\nu^{\alpha j} U_\nu^{\alpha k} U_\nu^{\beta j} U_\nu^{\beta k} \ . \qquad (33)$$

We see that the solar neutrino puzzle is explained by maximal vacuum oscillations of ν_e into $\nu_{\mu,\tau}$. For $b \sim c$ the $\nu_\mu - \nu_\tau$ mixing is naturally large, as suggested by the atmospheric anomaly. For $b \simeq c$ the $\nu_\mu - \nu_\tau$ mixing will be even maximal and ν_e state oscillations will be 50% into ν_μ and 50% into ν_τ. This bi-maximal neutrino mixing scenario, which we have realized in the framework of SUSY $SO(10)$ model, closely resembles the bi-maximal neutrino mixing suggested in ref. [3] for $SU(5)$ GUT.

One may wonder whether the small mixing angle MSW solution for the solar neutrino puzzle can be realized within our $SO(10)$ scheme. From (16) we see that the expected mixing between ν_e and $\nu_{\mu,\tau}$ states is $\sim \epsilon$, which is too large for the small angle MSW oscillations. This can be improved if we modify the $\mathcal{U}(1)$ charge of 10_1 state to $Q_{10_1} = 1/2$. The oscillation angle will then have the desirable value $\sim \epsilon^2$. With this modification the hierarchies between the down quark and charged lepton masses will still be reasonable, $m_{e,d}/m_{\mu,s} \sim \epsilon^3$. To obtain $\nu_e - \nu_{\mu,\tau}$ oscillation we can introduce a $SO(10)$ singlet state N (instead of $\mathcal{N}_{1,2}$ states), which will provide mass in the 10^{-3} eV range to the 'ν_2' state, so that the small angle MSW oscillation for explaining the solar neutrino deficit is realized.

As far as the large mixing angle MSW solution is concerned, by keeping the $\mathcal{N}_{1,2}$ states with the transformation properties in (20), maximal $\nu_e - \nu_{\mu,\tau}$ oscillations will still hold and the desired scale ($\sim 10^{-6}$ eV^2) can be generated by taking $\kappa' \sim 10^{-2}$ in (26). The oscillation picture (32) for the atmospheric neutrinos will be unchanged.

Before concluding, let us briefly discuss the question of nucleon decay within the proposed $SO(10)$ scheme. First of all, let us note that the operators $16_i 16_j \bar{C}\bar{C}'$ and $16_i 16_j \bar{C}\bar{C}$, which could induce the dominant decay modes [13] in $SO(10)$ are forbidden by the $\mathcal{U}(1)$ symmetry. This means that the states $\nu^c_{16_i}$ remain massless. However, this does not affect anything since they do not have 'Dirac'-type couplings with the 'light' ν_{10_i} states (the operators $10_i 16_j \bar{C} H$ are forbidden by $\mathcal{U}(1)$ symmetry).

The couplings qqT and $ql\bar{T}$ emerge from (14) and (16) respectively and take the form:

$$q\hat{Y}_U q T_H + q\hat{Y}_D l \bar{T}_{C'} \ , \qquad (34)$$

where Y_U, Y_D denote the Yukawa matrices of up and down quarks, and T_H and $\bar{T}_{C'}$ are color triplets from H and C' respectively. In order to build dimension 5 operators the triplet states must be integrated out. Taking into account equation (8) as well as the assumption that color triplets from H have masses of order M_G, the nucleon decay amplitude (for $p \to K\nu$) will be suppressed $\sim \frac{\epsilon_G}{M_P \epsilon^5} \simeq \frac{1}{5M_G}$, leading to a suppression by a factor $5 - 10$, relative to the minimal $SU(5)$ scheme. We therefore estimate the proton life time to be $\tau_{p\to K\nu}[SO(10)] \sim (10-100)\cdot \tau_{p\to K\nu}[\text{minimal } SU(5)]$. Hopefully, SuperKamiokande can observe such decays in the not too distant future!

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